THE STAINED GLASS WINDOW SUNDIAL

SCOTCH COLLEGE, MELBOURNE

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INTRODUCTION

The main administrative buildings at Scotch College were extended during the period, 1989 to 1990. To celebrate this occasion, a former student, Mr Clive Tingate, who attended the school from 1915 to 1918 very generously donated a most unusual gift. This gift took the form of a **Stained Glass Window Sundial**. It was proposed to install this sundial in one of the window frames of the staff room in the new building.

This sundial indicates correct solar time by means of a shadow cast on to the sand blasted part of the glass surface. The shadow is seen from inside the room and is produced by the shadow casting device (a gnomon) which is mounted externally on the window pane.

Most of us are already familiar with conventional, horizontal type, garden sundials[1]. These have been used for many centuries for telling the time by measuring the daily movement of shadows cast on to a horizontal plate.

In a similar fashion, vertical or wall sundials can be used for telling the time. Examples of these vertical dials can be seen around Europe and the Middle East on the walls of castles, stately homes and public buildings such as churches and town halls.[2]

A most unusual form of vertical sundial can be made as a stained glass window. A few of these rare sundials, dating from the middle of the 17th Century, still exist in the museums of Britain and Europe. Their rarity is such that only 32 examples are known in Britain and about 50 to 60 in the whole of Europe. These ancient time telling devices were generally very small in area (typically 150 x 150 mm) and occupied just a small portion of the total area of a stained glass window.[3]

Before proceeding further, and to give us a better understanding of sundials in general, and the stained glass window sundial in particular, we should consider the following:

GARDEN TYPE SUNDIALS

To help you understand how a sundial functions, **Figure 1** shows the parts of a sundial which will be referred to in the following text.

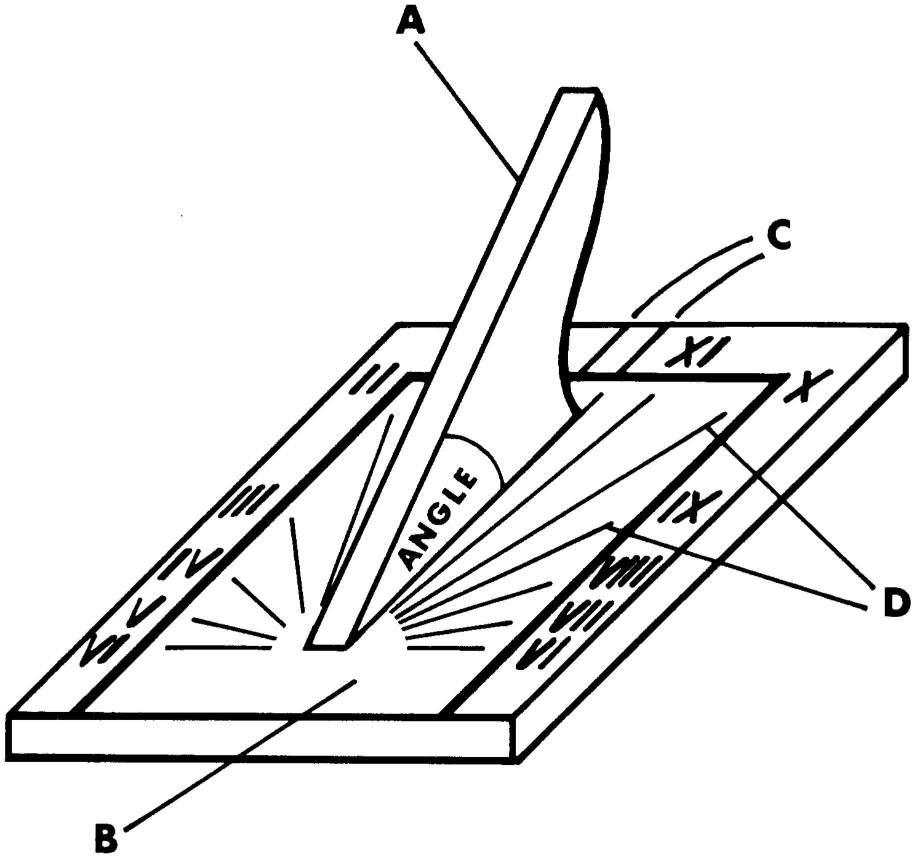


Figure 1: Parts of a conventional, horizontal, garden sundial

A The Style or Gnomon

This is the part that casts the shadow. Its upper surface must be parallel to the Earth's axis of rotation. To achieve this, the angle the gnomon makes with a horizontal dial plate must be the same angle as your latitude.

B The Dial Plate or Dial Face

This is the face onto which the shadow of the gnomon is projected by the Sun's rays. A set of lines and numbers positioned on this plate allow solar time to be determined.

For the horizontal sundial this part must be precisely horizontal.

C The Solar Noon Line or Lines

With the Sun directly overhead at Solar Noon for your location, the shadow of the gnomon will fall exactly between these lines. Note that if the gnomon had no thickness at all, the two Noon lines would coincide and become one line. However the gnomon must have some thickness for durability and therefore the distance between the Noon lines will be the same as the thickness of the gnomon. You may find that Solar Noon does not coincide with Clock Noon in your Time Zone. This will be explained later.

D The Hour Lines

When the shadow of the gnomon falls on one of these lines, then that is the time. You may also add half-hour lines or quarter-hour lines according to your wishes and within the limitations of the dial size.

SUNDIALS - DEFINITIONS AND BASIC TYPES

It is reasonable to suppose that from the earliest times mankind has used the apparently moving Sun as a means for reckoning time. As the Sun appears to move across the heavens during the day, the position and length of the shadow cast by a solid rod will continually change. The angular position or length of this shadow may be used for the purpose of subdividing the period between sunrise and sunset. Any device that uses the shadow cast by the Sun for the purpose of dividing the day into equal parts is known as a SUNDIAL.

Sundials came into general use in the 13th Century and various types were rapidly developed. By the time mechanical clocks and watches made their appearance in the 15th Century, many different types of sundial had been constructed and volumes written on the theory of the various devices.

There are two fundamental types of sundial: (i) those which measure the DIRECTION of the shadow cast by the Sun, and (ii) those which measure the LENGTH of this shadow. The most common fixed types of sundial mark the direction of the Sun's shadow by means of divisions on the dial plate. The dial or plate itself can be horizontal or vertical and the style or gnomon which casts the shadow is placed so that it is parallel to the axis of rotation of the Earth. In the Southern Hemisphere this means that the gnomon should be aligned along the true North-South line and inclined to the horizontal at an angle which is equal to the latitude, with the highest point of the gnomon nearest the South Pole. **Figure 2** illustrates the situation for a latitude of 35° South.

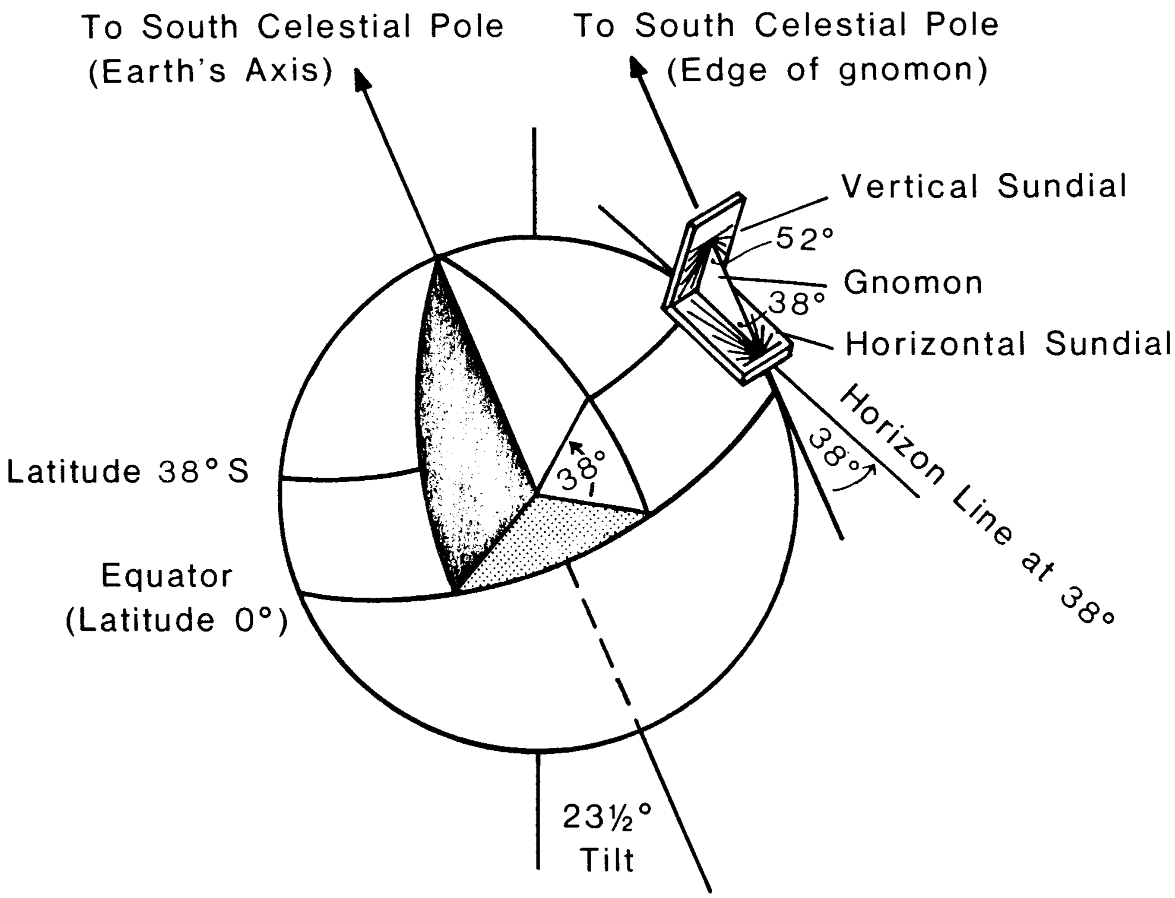


Figure 2: Sundial Location on Earth

As the Sun moves across the sky then the shadow of the gnomon moves across the scale around the dial plate. For measuring Solar Time the morning and afternoon hour lines are placed symmetrically about the Solar Noon line. The hour lines themselves are not equally spaced, and the angle between any adjacent hour lines depends on the observer's latitude.

Reclining or Vertical dials, as defined in **Figure 3**, can be constructed in a similar fashion, and provided that the gnomon faces due South in the Southern Hemisphere, then the hour lines will be positioned symmetrically about the central gnomon, whose position constitutes the Noon line.

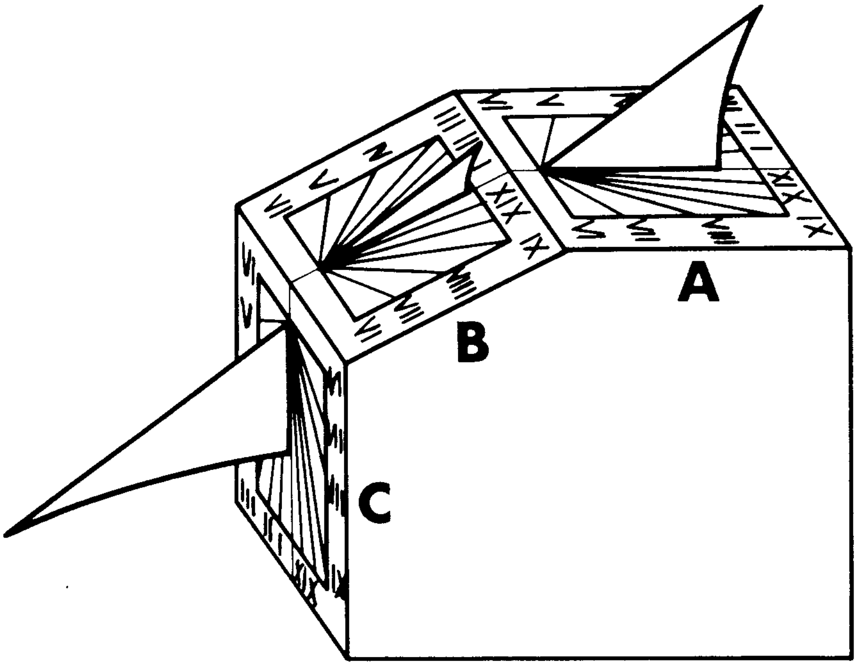


Figure 3: Surface A takes a horizontal dial, surface B a reclining dial, and surface C a vertical dial

However, if a vertical dial is fitted to a wall which does not lie in a direct East-West line (such a sundial is said to be "vertically declining"), then the hour angles will no longer be symmetrically placed about the Noon line, and the degree of asymmetry will increase as the deviation of the wall from true East-West increases.

The sundials described so far tell the time by defining the angular movement throughout the day of the shadow cast by the upper edge of the gnomon onto either a horizontal or vertical surface. i.e. they find the time from the direction of the Sun's shadow.

It is also possible to find the time by measuring the Sun's altitude, or rather the length of a shadow. The simplest case is the vertical shadow stick, whose shadow is shortest at Noon and longest at sunrise and sunset. However, because of the changes in position of the Sun with season, it is necessary to have different scales of time for different periods of the year. These difficulties often result in simple sundials which only tell the time approximately, and such sundials are now considered merely as ornaments or items of historical curiosity.

However, if properly constructed, sundials which measure the length of the Sun's shadow can indeed be very accurate. A good example is the Pillar or Shepherd's dial. This class of sundial differs fundamentally from those described earlier in that the gnomon does not point to the celestial pole. Instead the gnomon is horizontal and can rotate about a vertical column on which are inscribed a series of spiral curves representing the Sun's altitude at different times throughout the year, and vertical lines representing the months. The gnomon is set at the position representing the date and pointed at the Sun. When the shadow is exactly vertical then the length of the shadow indicates the time, read between the various spiral curves.

TIME KEEPING ACCURACY OF SUNDIALS

The biggest problem with sundials purchased over the counter in garden shops is that they are never, except by the purest accident, designed for the location at which they are to be used. This has led to the idea that sundials do not tell the time correctly. Nothing could be further from the truth; when properly designed for their location, sundials can readily give readings accurate to one minute or better.

In adjusting the horizontal or garden type sundial, it is important to remember that the gnomon should be parallel to the axis of the Earth's rotation. That is, it should lie exactly in the true North-South plane, with the South end so elevated that the angle which the gnomon makes with the horizontal plane is equal to the latitude of the observer's location. When properly adjusted, the sundial will read Solar Time (Local Apparent Time) which is not the same as the time indicated on a clock.

RELATING SUNDIAL TIME TO CLOCK TIME

To obtain the same time as indicated on a clock, two corrections must be made to the time indicated by the shadow on a simple symmetrical sundial. The first correction is caused by the varying distance of the Earth from the Sun, while the second is due to the setting up of Standard Times Zones over the Earth's surface.

First Correction

The Earth's orbit is almost circular, but the Sun is somewhat off-centre and so we are further away from the Sun on July 1 (152,080,000 km) than we are on January 1 (146,930,000 km). Kepler discovered four centuries ago that the Earth slows down as we swing further away from the Sun, and speeds up as we draw nearer. This variation gives rise to a difference between Local Noon as given by a man-made clock which keeps time uniformly (based on the motion of the Sun averaged over a whole year) and Solar Noon which occurs when the Sun is directly overhead at a given location. This correction factor is the same everywhere on Earth, and to convert Sundial (shadow) Time to the Local Time, the Equation of Time correction shown in **Figure 4** must be added to the shadow reading.

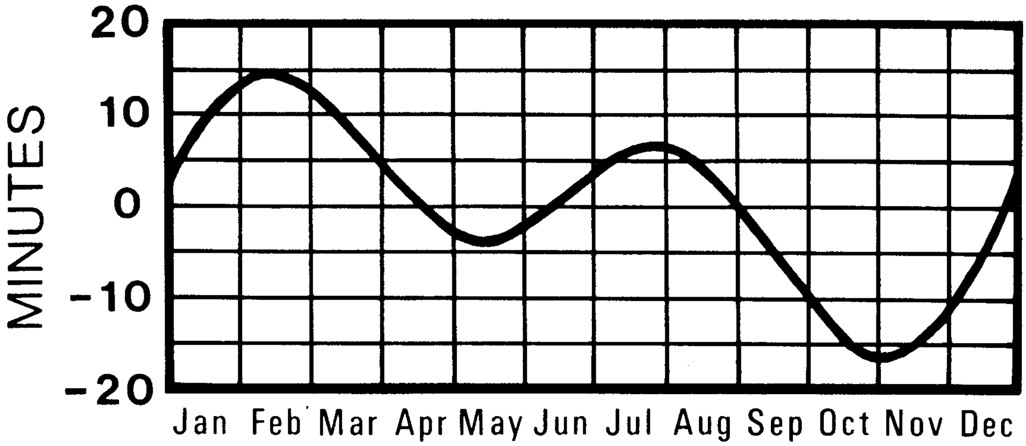


Figure 4: Equation of Time Correction

Second Correction

The Earth rotates daily about its own axis in an eastwards direction and the sun appears to travel one complete revolution of 360° of longitude in every 24 hours of time.

During the year, the Earth rotates around the Sun in an orbit called the ECLIPTIC. The plane of the orbit is inclined at a fixed angle of 23.5° to the plane of the equator of the CELESTIAL SPHERE. This is shown in **Figures 5 and 6** below. (A more detailed description of the celestial sphere will be given later on in this report.)

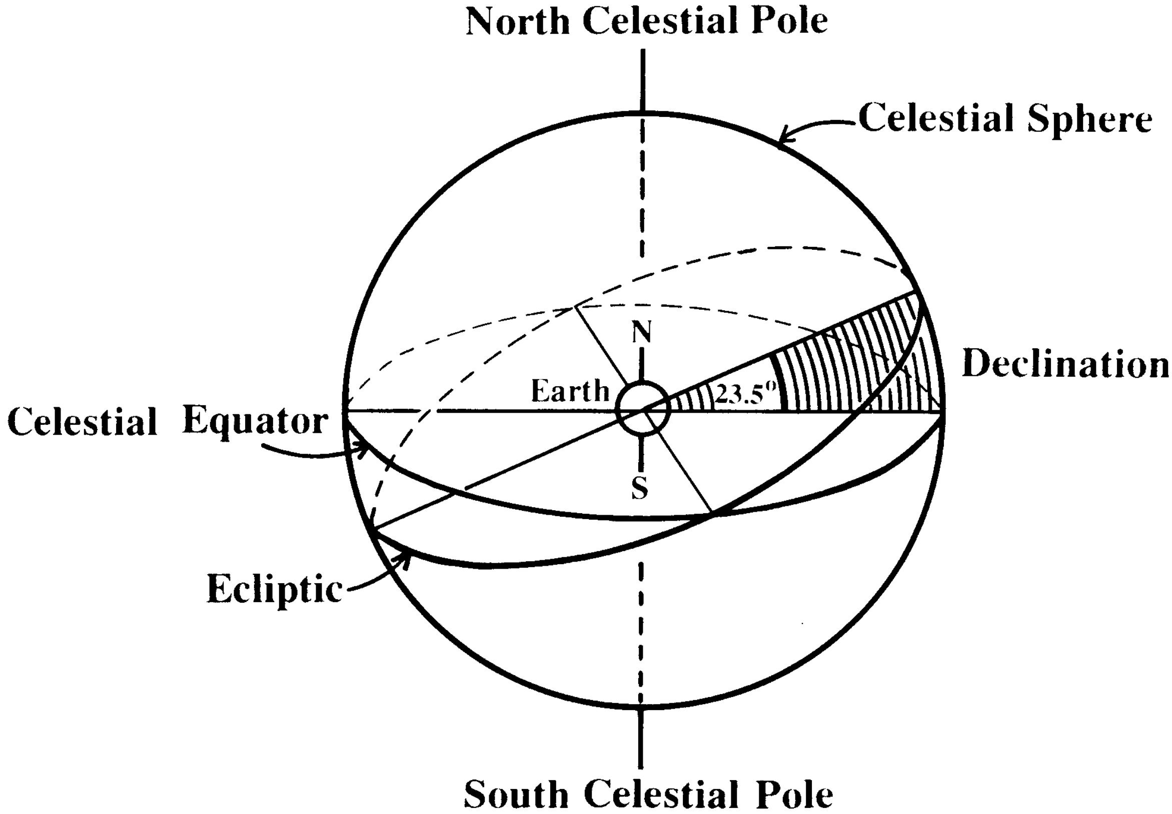


Figure 5: The Celestial Sphere

Whilst in orbit around the sun, the Earth is simultaneously rotating about its own axis. The effective latitude angle of the Sun, known as the DECLINATION, on the celestial sphere, will thus vary throughout the year and there will be a corresponding change in the sunrise and sunset times for every place on Earth.

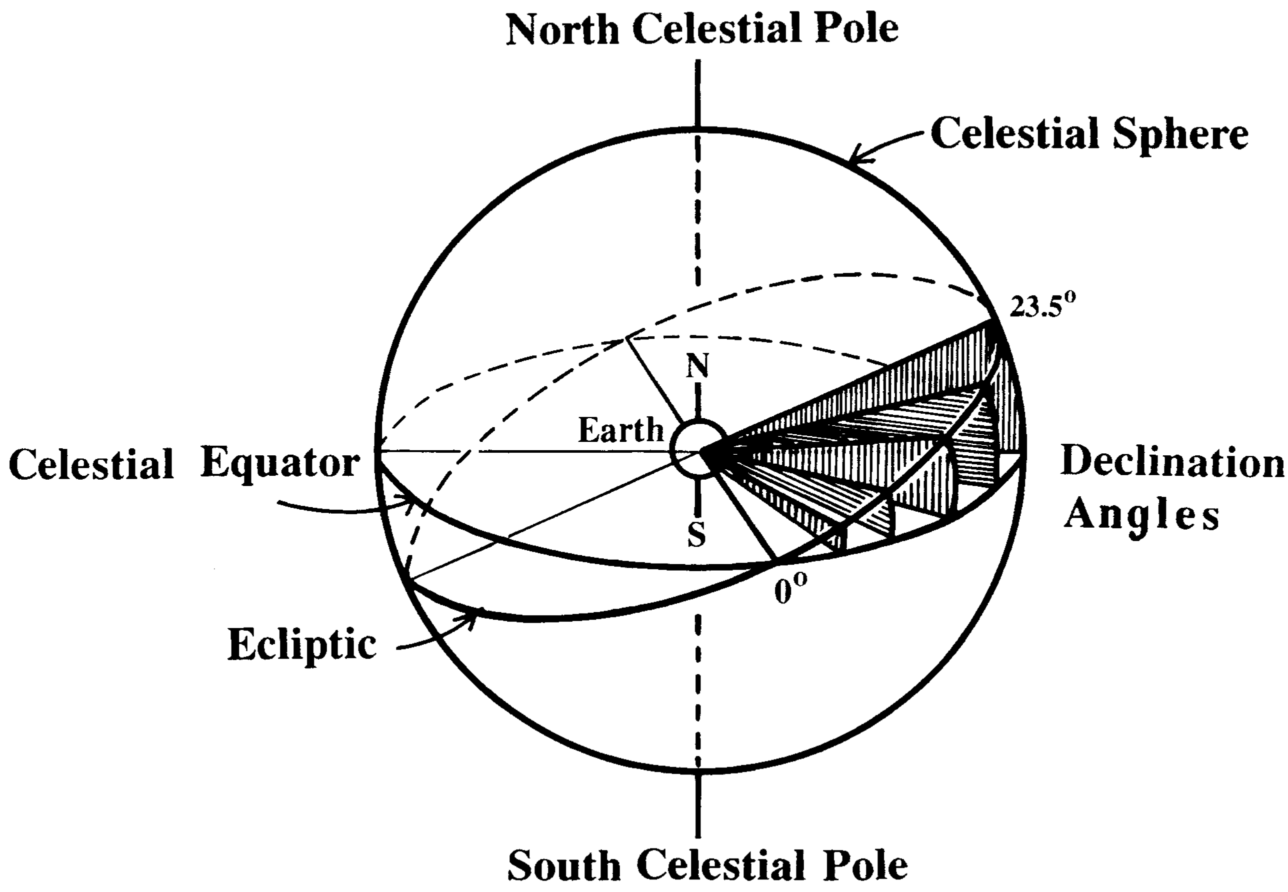


Figure 6: How Declination Varies Throughout the Year

Every place will have its own LOCAL TIME which differs from every other place East or West of it (but not North or South of it, because the Sun strikes all places on the same longitude, at the same time). Local Time will differ by 4 minutes of clock time for any two places on Earth which are separated by 1° of longitude. For every place to have a different time would be too complicated, and so for convenience the world is basically divided into 24 times zones set 15° of longitude apart, each representing a difference in time of one hour. In practice however, the zone boundaries are altered here and there to better fit in with the boundaries of countries, islands etc. Throughout each zone the time is constant at the Zone Standard Time, and differs from the Standard Time in adjacent zones. Specific locations within a particular time zone will show actual Solar Noon some minutes before or after the Standard Noon for that time zone. The Local and Standard times may differ appreciably at the outer limits of the zones.

Zone (Clock) Time = Local Time + Correction for longitude displacement of the sundial East or West of the Zone Standard Meridian.

Note that the longitude time correction for a given location is constant throughout the year, and depends only on the longitude difference between the Standard Meridian and the observation point.

Examples	**Perth**	**Sydney**	**Melbourne**
Latitude	31°57'S	33°53'S	37°49'S
Longitude	115°51'E	151°13'E	144°58'E
Longitude of Time Zone	120°00'E (Western Standard Time Zone)	150°00'E	150°00'E
		(Eastern Standard Time Zone)	
Longitude Difference	- 4°09'	+ 1°13'	- 5°02'
Time Correction	+ 16.60 min.	- 4.87 min.	+ 20.13 min.

i.e. The Time Correction indicated above must be added to the Local Time to obtain the same time as indicated by a clock (Zone Time).

Resultant Time Correction

The apparently complicated final result is shown below.

Zone Clock Time = Time indicated by Sundial shadow + Equation of Time Correction + Longitude Correction for Standard Zone

i.e. Clock Time = Time indicated by Sundial shadow + Resultant Time Correction

The Resultant Time Correction may readily be incorporated onto the sundial face in either graphical or tabular form, so that a known number of minutes is added to or subtracted from the shadow reading to give directly the Zone Standard Time. This is shown in **Figure 7** for Melbourne.

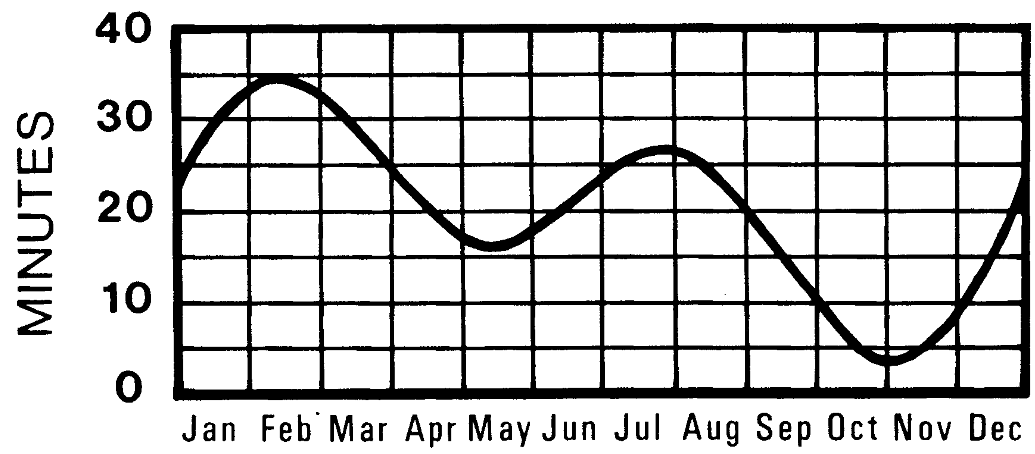


Figure 7: Resultant Time Correction for Melbourne
(To be added to shadow reading)

SUMMERTIME

During the summer, it is common practice for different states and countries to add varying amounts of time to zone standard time in order to extend the hours of available evening daylight. This Summertime correction, usually one hour, should be added to sundial time where necessary.

For example, Victorian Summertime extends from late October until early March. During this period, Eastern Summertime is one hour ahead of Eastern Standard Time. Thus if your Sundial reading + Equation of Time correction + Longitude correction reads, say, 4.15 p.m. Eastern Standard Time, then you must add a further one hour to obtain the same reading as your watch, 5.15 p.m. Eastern Summertime.

HOW THE STAINED GLASS WINDOW SUNDIAL WAS MADE

Designing, Computing and Painting

Prior to making any vertical wall or window type sundial, it is essential to know the angular deviation of the wall from a true East-West line.

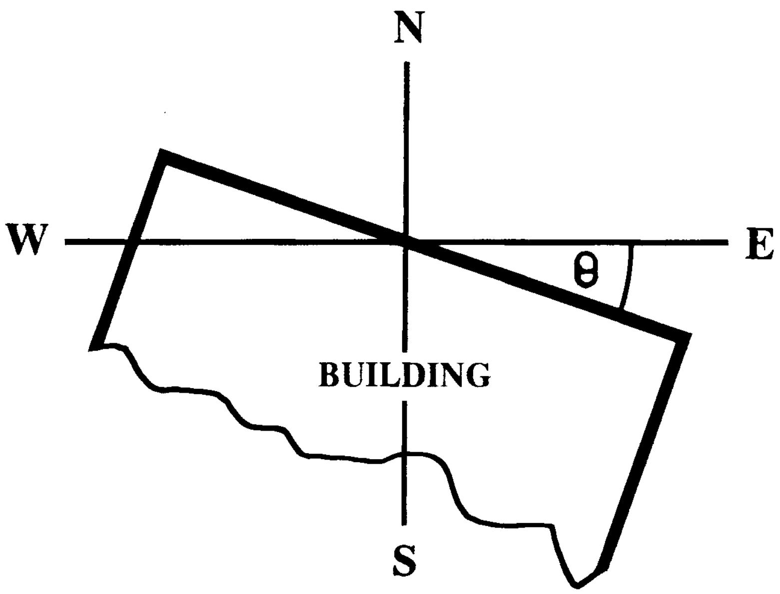


Figure 8: Angle of Deviation from a True East West Line

There are several ways of doing this as described by R. Newton Myall in his book, 'Sundials'[1].

The required angle 'θ' was measured by a local surveyor and found to be very closely zero. Next, the accurate latitude and longitude of the College were determined from a recently published ordnance survey map.

These measurements then enabled calculations to be made which gave the required relations between the hour angle positions and the vertical line of solar noon.

The equation used to produce these angles is given on Page 18 of this report. In the special case when the wall lies precisely along the true East-West direction, then the angle θ will be zero. The resultant sundial will have hour angle positions which are symmetrical about the Solar Noon position. This is the case for the Scotch College sundial. See **Figure 9.**

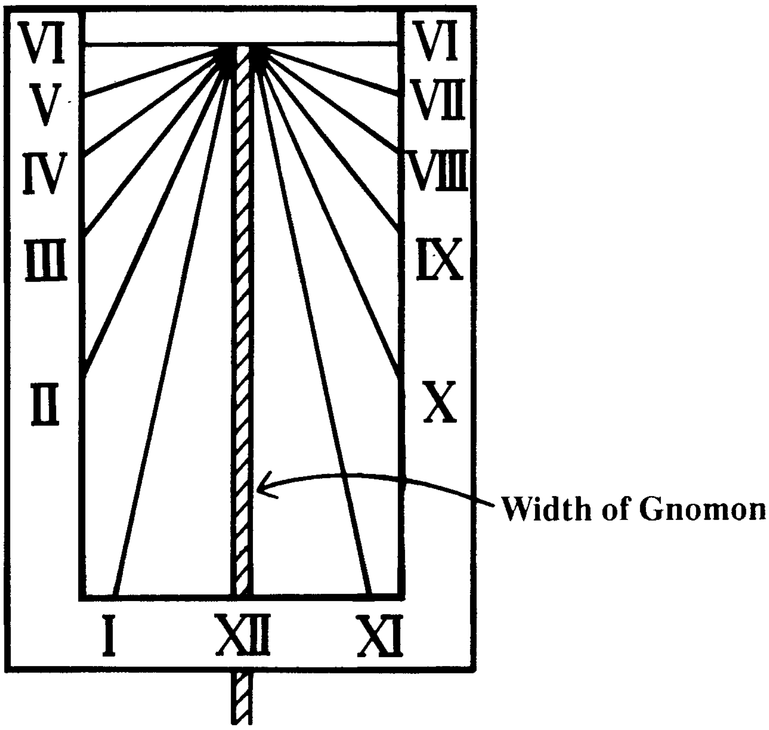


Figure 9: Symmetry of Hour Divisions about Solar Noon for a North Facing Wall or Window

Both whole hour and half hour divisions were then drawn out full size (1070 x 715 mm window frame) and the old and new school coats of arms were added. At this stage, discussions were entered into with the College to ensure authenticity of words, colour, proportion and accuracy. Finally, a large black and white line drawing was produced as the starting point or 'cartoon' for the stained glass window sundial. This master drawing was now correct with respect to the required sundial specifications and accuracy. The College also confirmed that, overall, the sundial design was aesthetically pleasing and it satisfied their requirements for colour, proportion, texture and layout.

A local ceramic artist, Jan Milton, was asked to paint the required numbers, letters and designs onto the different shaped pieces of glass. She used finely powdered glass, called frit, which was mixed with water glass (sodium silicate) prior to firing in an oven. To obtain the required intensity of colour it was frequently necessary to apply many coats of this glass paint and fire each coat into position. Quite a tedious process but one which will certainly assure the longevity of this painted sundial.

Figure 10: **Jan Milton the Ceramic Artist, Painting the Original College Coat of Arms onto a Glass Plate**

ASSEMBLING THE GLASS SUNDIAL AND GNOMON

A well known lead-light window maker and teacher, Ron Sutton, was responsible for cutting and assembling the sundial window.

Figure 11: Ron Sutton the Glazier Assembling the Stained Glass Window Sundial

Cathedral quality glass was used throughout and the lead cames surrounding the various pieces of coloured and painted glass were very carefully soldered to each other to give strength and durability. To give extra strength with respect to wind loading, and as a secure foundation for attaching the gnomon, a steel insert was fitted into the horizontal came adjacent to the semi-circular sun.

The cames were tightly packed with whiting (a mixture of finely powdered chalk and raw linseed oil) to resist water over the centuries to come. The whole glass window was surrounded by a strong brass U-section bar to give strength and support. Sometime in the future when the meranti timber frame has rotted away, it will be possible to remove the entire sundial window and replace it in another frame. The window itself was cemented to the wooden window frame with silastic cement.

THE GNOMON

The minimum length of the gnomon (see the Mathematics of Sundials section of this report) was about 750 mm to ensure that its shadow reached the lowest horizontal part of the time scale at solar noon on mid-winters day. At all other times throughout the year the shadow would always be longer than this.

The gnomon itself was made from glass surrounded by U-section lead came. In turn, the came forming the uppermost surface of the gnomon was soldered to a 10 mm diameter circular section, hard drawn copper rod. The final embellishment was a small copper ball which was screwed on to the end of the gnomon itself. This rod was later 'tinned' with solder then coated with a thin colouring layer of 'ZEBO' grate polish to give an antique finish.

The gnomon with its surrounding metal border and rigid upper edge, was then soldered to the appropriate portion of the glass sundial window. It was additionally supported with a 6.5 mm diameter, hard drawn copper rod, which straddled the gnomon and was then screwed into the window frame itself.

The angle of the gnomon where it was attached to the plane of the sundial was made equal to the CO-LATITUDE of Scotch College, (i.e. $90^\circ - \phi$), where ϕ is the latitude angle.

Figure 12: The Glass Gnomon. The Upper Shadow Casting Edge is 750 mm Long

DECORATIVE SYMBOLS ON THE SUNDIAL

There are three small symbols on the stained glass window sundial which have been fired on in an oven. They are:

1. A fly in the bottom left hand corner.
2. A rose in the bottom right hand corner.
3. An hour glass at the top of the dial, in the centre.

What do they signify?

A fly was often used as the symbol of a master craftsman in window making from about the 12th Century onwards to the present day. It was necessary to have served an apprenticeship, worked as a journeyman and practised as a tradesman before becoming a master glass craftsman.

A rose is the symbol used by Jan Milton, the ceramic artist, on all of her major works of art.

An hour glass is the symbol used by physicists Margaret Folkard and John Ward on their precision sundials.

THE COMMEMORATIVE PLAQUE

The bronze plaque layout shown in **Figures 13** below will be attached to the wall immediately adjacent to the stained glass sundial.

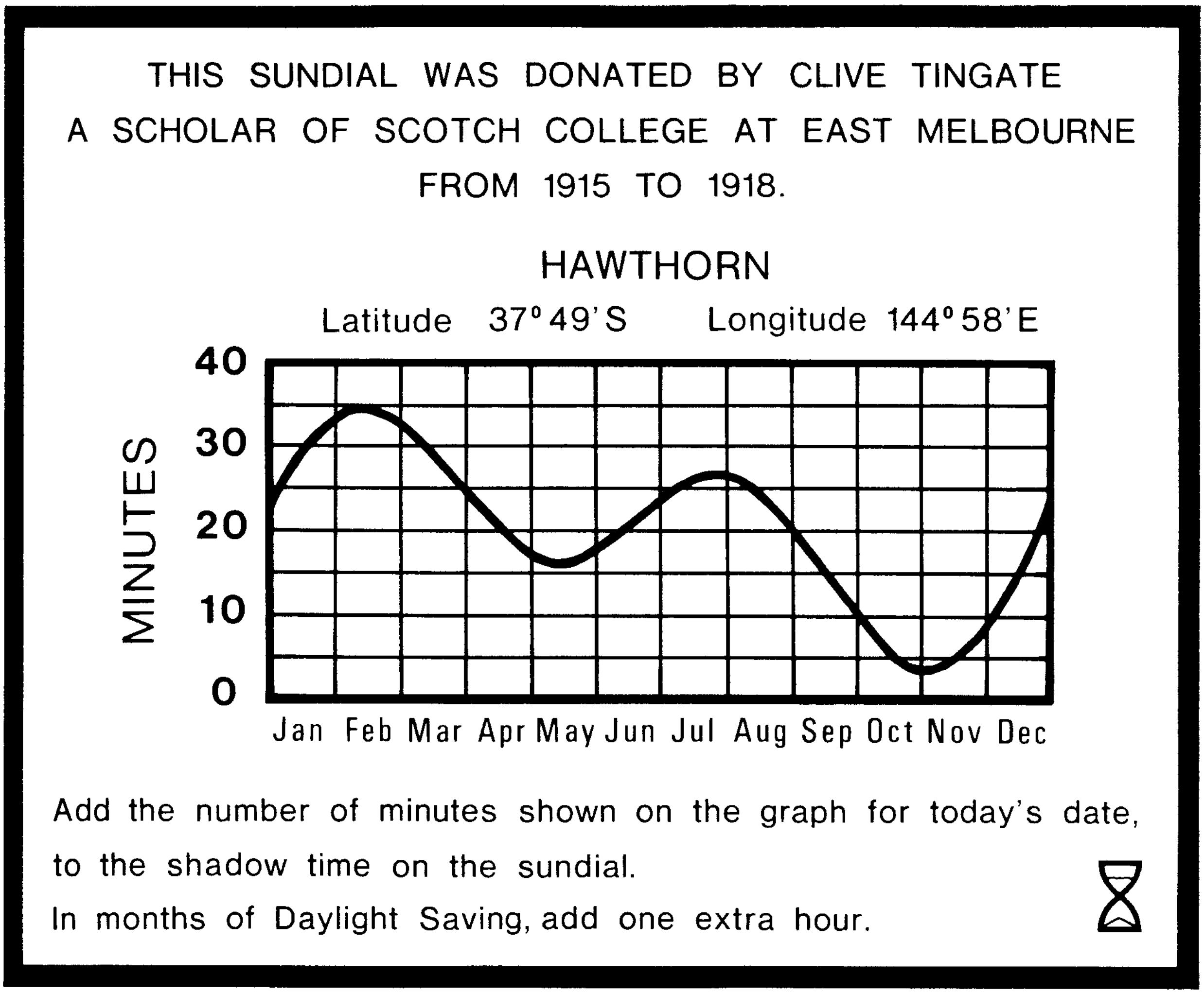


Figure 13: **The Layout of the Bronze Plaque Mounted on the Wall Adjacent to the Stained Glass Window Sundial 1990**

The curve shown on the commemorative plaque combines the Equation of Time with the difference in time between Melbourne at longitude 144°58'E and the longitude of the Eastern Standard Time (EST) zone located at 150°E. This procedure was explained in an earlier section of this report.

HOW TO TELL THE TIME
WITH THE STAINED GLASS WINDOW SUNDIAL

The following procedure should be used for telling the time:-

1. Stand centrally in front of the sundial window and ensure that your line of sight viewing position, is at right angles to the plane of the window and the wall it is mounted on.

2. The viewing distance from the sundial window should be comfortable for each observer. The distance is not in any way critical.

3. Next, you must carefully observe the position of the shadow on the hour scale of the sundial. In the morning, the shadow will be on the left hand side of the dial and in the afternoon, it will be on the right hand side.

4. Remember that it is only the **uppermost portion of the shadow** demarcation line which tells the time. This shadow boundary line can be followed with your eye until it crosses the green glass boundary containing the half hour divisions of time. Estimate the fraction of the half hour division.

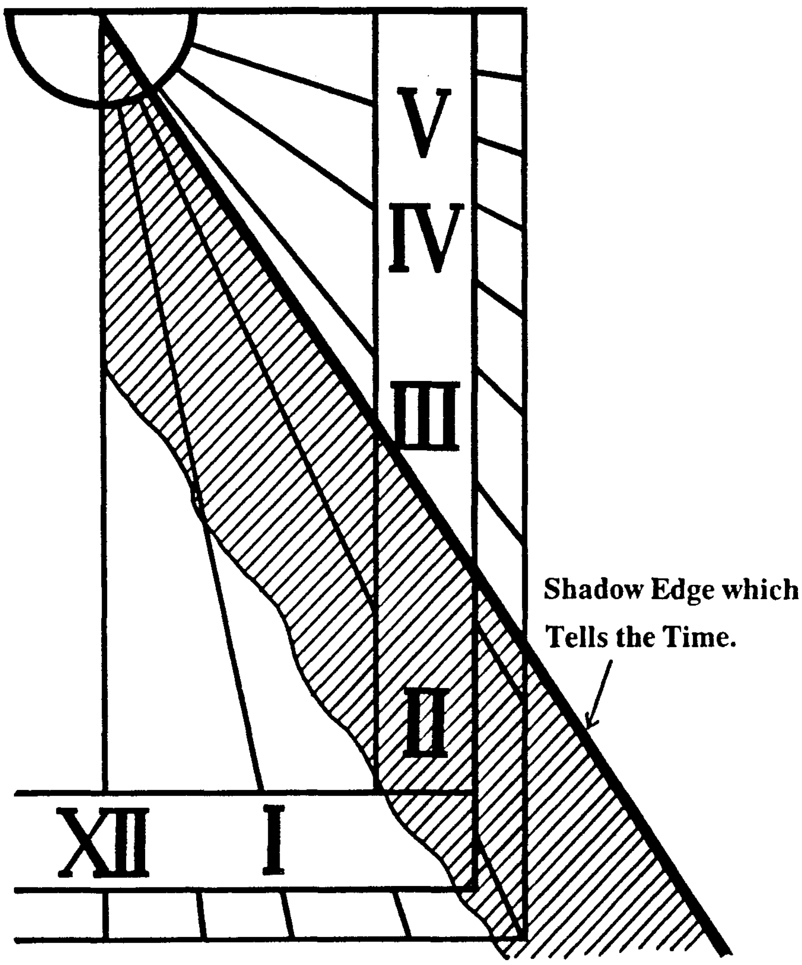


Figure 14: The Shadow Edge Which Tells the Time

5. In **Figure 14** above, the solar time shown by the shadow is 2.40 p.m.

6. To convert this shadow reading from solar time to the time shown on your watch (Eastern Standard Time, EST) you must now refer to the Time correction plate which is attached to the wall adjacent to the sundial.

For Example: In the middle of May, the correction is 16 minutes. Adding this to 2.40 p.m. gives 2.56 p.m. which is very closely the same time within a minute or two as that shown on your watch.

As the shadow position moves around the time scale throughout the day, its width will change because the gnomon is large. Early in the day the shadow width will be largest and it will decrease as the time approaches solar noon. At this time, the shadow width will be the same as that of the gnomon, i.e. 10 mm.

Some notes about shadows have been included in the section of this report entitled Mathematics of Sundials.

MATHEMATICS OF SUNDIALS

Many years ago the mathematics of sundials (gnomonics) was considered to be an essential part of a gentleman's education. Universities in Britain, especially Oxford and Cambridge, have college walls, windows and quadrangles which are adorned with many different types of sundials. Tedious arithmetical and graphical solutions based upon fundamental equations for calculating the altitude and azimuth of the sun, were invariably used. In our modern era this has all been replaced by powerful pocket calculators and desk top computers.

Using such a machine, it is easy to calculate the positions of the hour line shadows for any given location whose latitude and longitude are known. The basic equations for doing these computations are shown below.

A For a HORIZONTAL SUNDIAL

$$\tan \gamma = \sin \phi \tan (H)$$

where,

γ = angle of gnomon or style from solar noon.

ϕ = latitude angle

H = Hour angle, 1 hour = 15°
2 hours = 30°
.
.
.
6 hours = 90° etc.

B For a VERTICAL SUNDIAL

On a vertical wall lying East-West and facing due North or South.

$$\tan \gamma = \cos \phi \tan (H)$$

C For a VERTICAL SUNDIAL on a wall displaced by an angle 'θ' from the East-West Plane

Consider the following drawing:-

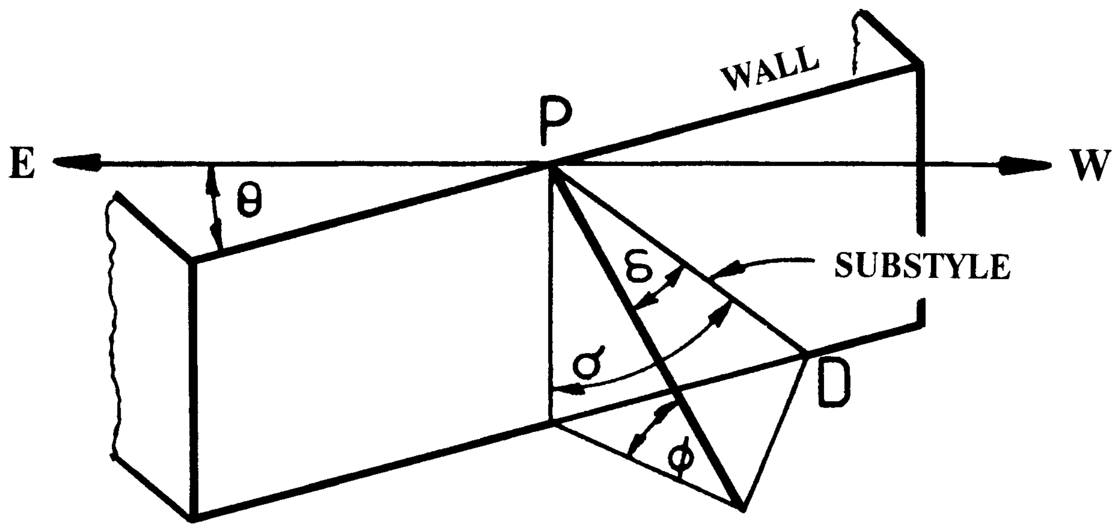


Figure 15: Vertical Sundial When the Wall is Not East-West

$$\tan \sigma = \frac{\sin \theta}{\tan \phi}$$

where θ = Angle of deviation from true East West line

ϕ = Latitude angle

σ = Angle of style from vertical noon line

$\sin \sigma = \cos \phi \cos \theta$

Let $\sin h = \dfrac{\sin \sigma}{\cos \phi}$

$\tan \gamma = \tan H \cos \phi$

where H = Hour angle of sun (15°/hour etc.)

n.b. H is positive in the forenoon in the Southern Hemisphere

H is negative in the forenoon in the Northern Hemisphere

finally we get,

$$\tan \gamma = \tan (h \pm H) . \cos \phi . \cos \theta$$

By substituting the appropriate latitude angle (ϕ) and the required hour angle (H) in degrees, the angular relationship γ between solar noon and any given hour (or fraction of an hour) can soon be calculated.

The total time correction needed to convert solar (sundial) time to Standard Time is found by adding the longitude correction (at the rate of 4 minutes of time per 1° of longitude difference) to the Equation of Time. This is described earlier in this report.

Typical results for the calculation of a horizontal sundial are shown in **Figure 16**.

SHADOWS, BLACKNESS AND EDGE SHARPNESS

If you look at the shadow cast by the sun from a vertical stick, or from the edge of a building, you will notice that the shadow boundary is sharpest and blackest at the point where the stick or wall meets the ground.

As your eye moves along the shadow line away from this point, the edge becomes increasingly fuzzy and the apparent difference between the blackness of the shadow and the background decreases. This difference is called contrast and it is an essential factor which has to be considered in the design of this type of sundial.

When you look at the ground just behind the stick and in the blackest part of the shadow you can easily see lots of detail. The black is therefore not completely black. On a cloud free day about 20% of the sun's light is transformed into scattered light. It is this scattered light which enables us to see the shadow details.

In addition to the scattered light in the sky there is another component which contributes significantly to the lack of shadow edge definition and the low contrast produced by placing opaque objects in front of ground glass screens. This component is caused by the DIVERGENCE ANGLE OF THE SUN.

The sun is not a point source of light. This means that the light it emits does not emerge as parallel beams of light. Its divergence angle is about 0.5°. This sounds very small, however, if we look at **Figure 17** we can soon see that serious changes in contrast, focus and energy distribution can and do take place under almost every condition of shadow forming when using the sun as a light source.

In **Figure 17** let an opaque rod of diameter 2a be located a distance 'd' from a horizontal surface AC in a two dimensional plane. If the sun is incident on the rod at an angle 'α', then we can use geometry to deduce the following (the solution given is not exact in its present form but is sufficiently close for this purpose)

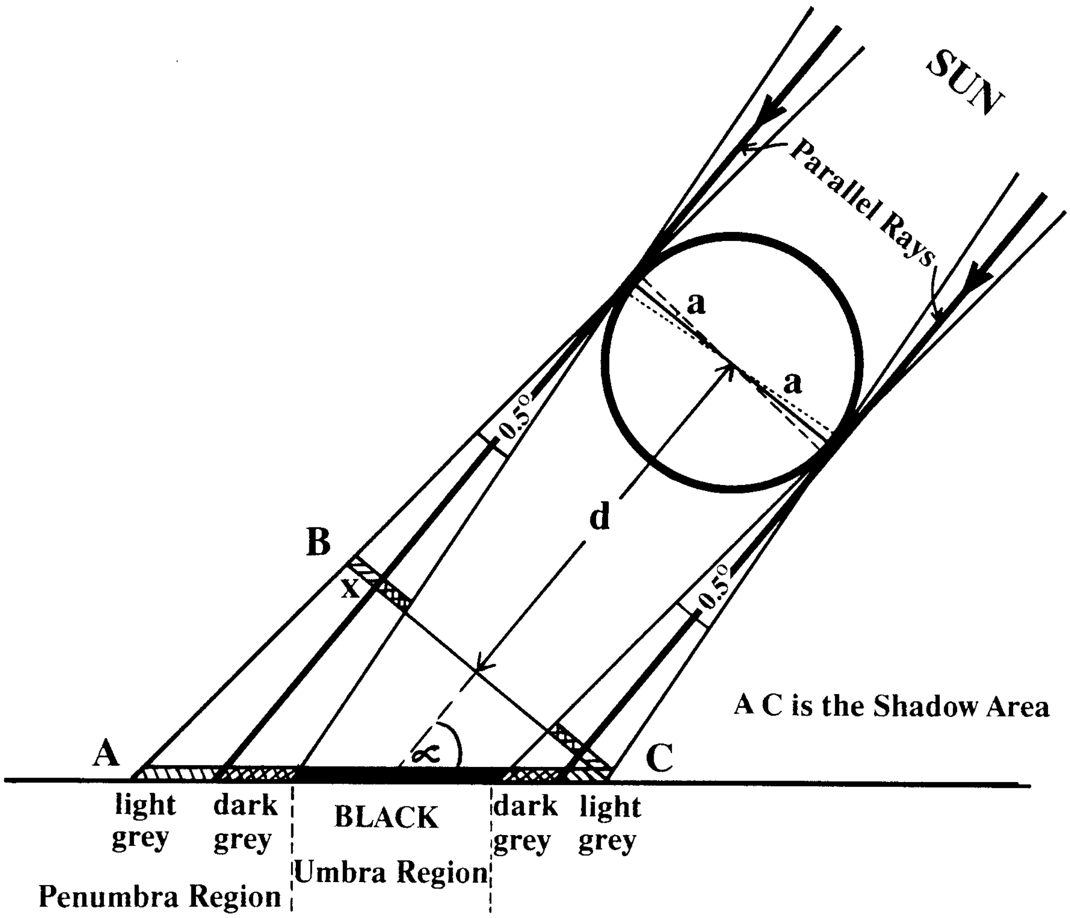


Figure 17: Shadows and Edge Sharpness

$x = d\tan(0.25)$

$BC = 2(a + d\tan(0.25))$

$\hat{BAC} = (\alpha - 0.25)$

$\hat{ABC} = (90 + 0.25)$

In Δ ABC, using the sine rule, we get,

$$\frac{BC}{\sin BAC} = \frac{AC}{\sin ABC}$$

$$\text{or} \quad AC = BC\frac{\sin \hat{ABC}}{\sin \hat{BAC}} = BC\frac{\sin 90.25}{\sin(\alpha - 0.25)}$$

Substituting BC gives,

$$AC = 2(a + d\tan 0.25)\frac{\sin 90.25}{\sin(\alpha - 0.25)}$$

Substituting some values:

Example 1

Let $a = 5$ mm

$d = 500$ mm

$\alpha = 90°$

$$AC = 2(5 \text{ x } 500 \tan 0.25) \frac{\sin 90.25}{\sin 89.75}$$

Therefore: AC = 14.36 mm

Example 2

Let $a = 5$ mm

$d = 500$ mm

$\alpha = 45°$

$$AC = 2(5 + 500 \tan 0.25) \frac{\sin 90.25}{\sin 44.75}$$

Therefore: AC = 20.40 mm

From the above results we can say that the shadow of a rod will become wider when we increase its distance from a ground glass screen and the light or energy distribution in the shadow area will vary considerably thus producing a less black or lower contrast image.

This explanation is not intended to be a rigorous solution to the problem of shadow formation, rather it is intended to produce an awareness of some of the difficulties which are inherent in shadow forming devices such as the Scotch College, Stained Glass Window Sundial.

LENGTH OF GNOMON NEEDED FOR THE STAINED GLASS WINDOW SUNDIAL

To ensure that the shadow of the gnomon always reaches the time scale throughout the year, it is necessary for the gnomon to be longer than a specified minimum length.

The gnomon shadow will be shortest when the sun is low in the sky and longest when the sun is high in the sky. On any day in the year, the sun is always at the highest point in the sky at SOLAR NOON. **Figure 18** below shows the situation.

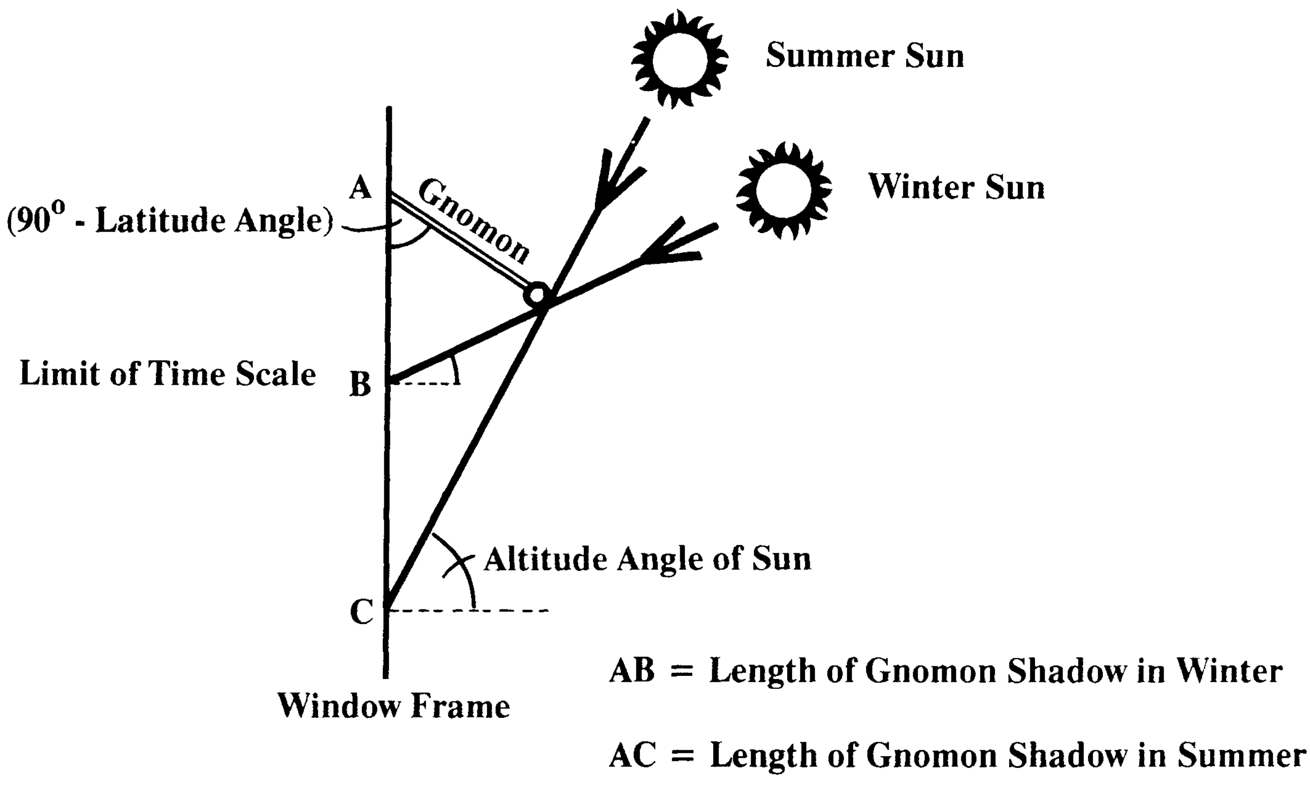


Figure 18: Variations of Gnomon Shadow Length at Solar Noon

At solar noon the shadow of the gnomon will always be seen as a vertical line which extends downwards from the point of attachment of the gnomon. At any other time of the day, the gnomon's shadow length will always be longer than that produced at solar noon. The detailed explanation of this statement is beyond the scope of this report. We will therefore concern ourselves with determining the minimum length of the gnomon required for the Scotch College Sundial.

Before we can calculate this length, consider **Figure 19**. It shows diagrammatically how the Earth travels throughout the year in its elliptical orbit around the sun (more precisely, the sun is located at one of the two foci of the elliptical orbit). The Earth itself rotates daily on its own axis and takes very closely 24 hours to do so. Simultaneously with this rotation, the Earth is in orbit around the Sun.

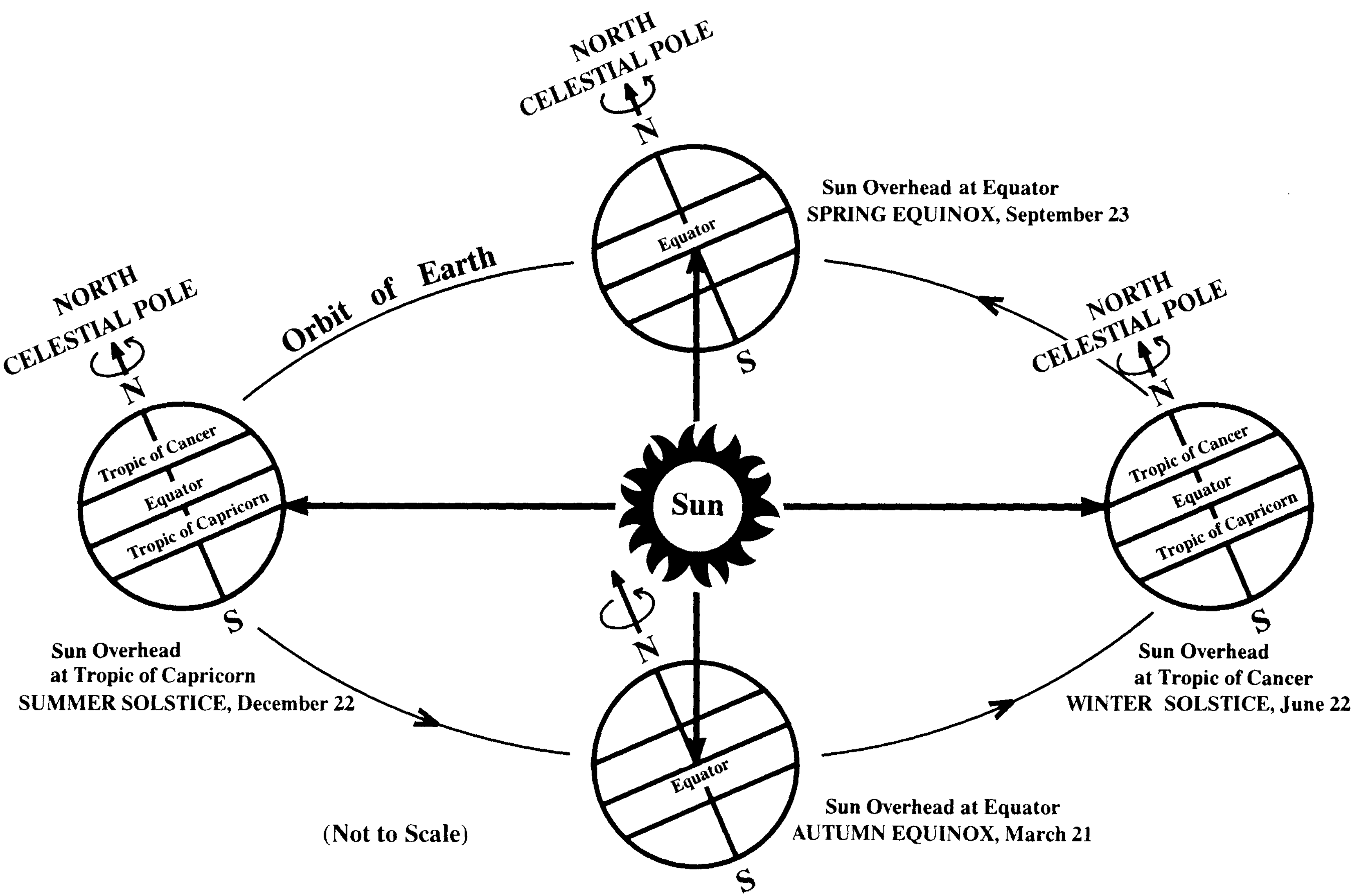


Figure 19: Orbit of the Earth Around the Sun and the Seasons for the Southern Hemisphere

To achieve a better understanding of these different motions we must use the concept of the CELESTIAL SPHERE. Since the dawn of history the concept has persisted of the heavens being a sphere on to which all the so called 'fixed stars' are projected. This celestial sphere may be defined as a sphere of infinite radius with the centre located within our solar system. The reference frames for all systems of astronomical, spherical co-ordinates are based on the celestial sphere. When projecting different members of the solar system onto the celestial sphere it is necessary to restrict the location of the centre to some particular point within the solar system.

If the centre of the celestial sphere is considered to be a point on the surface of the Earth, then we have systems of **apparent co-ordinates.** If the centre is at the centre of the Earth we have **geocentric co-ordinates,** if at the centre of the sun, **heliocentric co-ordinates** etc.

Because the Earth is rotating about its own axis, the celestial sphere is apparently rotating (as seen from the Earth) about an axis parallel to the axis of the Earth and with the same angular velocity as the Earth, but of course in the opposite direction. For convenience, this apparent rotation is referred to as the rotation of the Celestial Sphere. All reference systems with spherical co-ordinates that are established on the sphere, such as the equatorial, galactic, ecliptic etc., rotate with the sphere.

Of particular interest in this case, is the ECLIPTIC. This is the great circle cut out on the celestial sphere by the plane containing the orbit of the Earth. It is the fundamental plane for the system of spherical co-ordinates in which latitude and longitude are measured. During one half of the year, the sun appears to be north of the celestial equator and during the other half, south of it. The sun's angular distance from the equator is called its DECLINATION. It is measured in degrees and minutes and varies from day to day throughout the year. Similarly. lines running north to south are called lines of RIGHT ASCENSION. Thus lines of Right Ascension and Declination provide a reference system for all celestial objects, just as terrestrial longitude and latitude provide a reference system for all planes, ships or air craft which are moving over the surface of the Earth.

The ecliptic is also the reference plane to which the planes of the orbits of all the members of the solar system are referred.

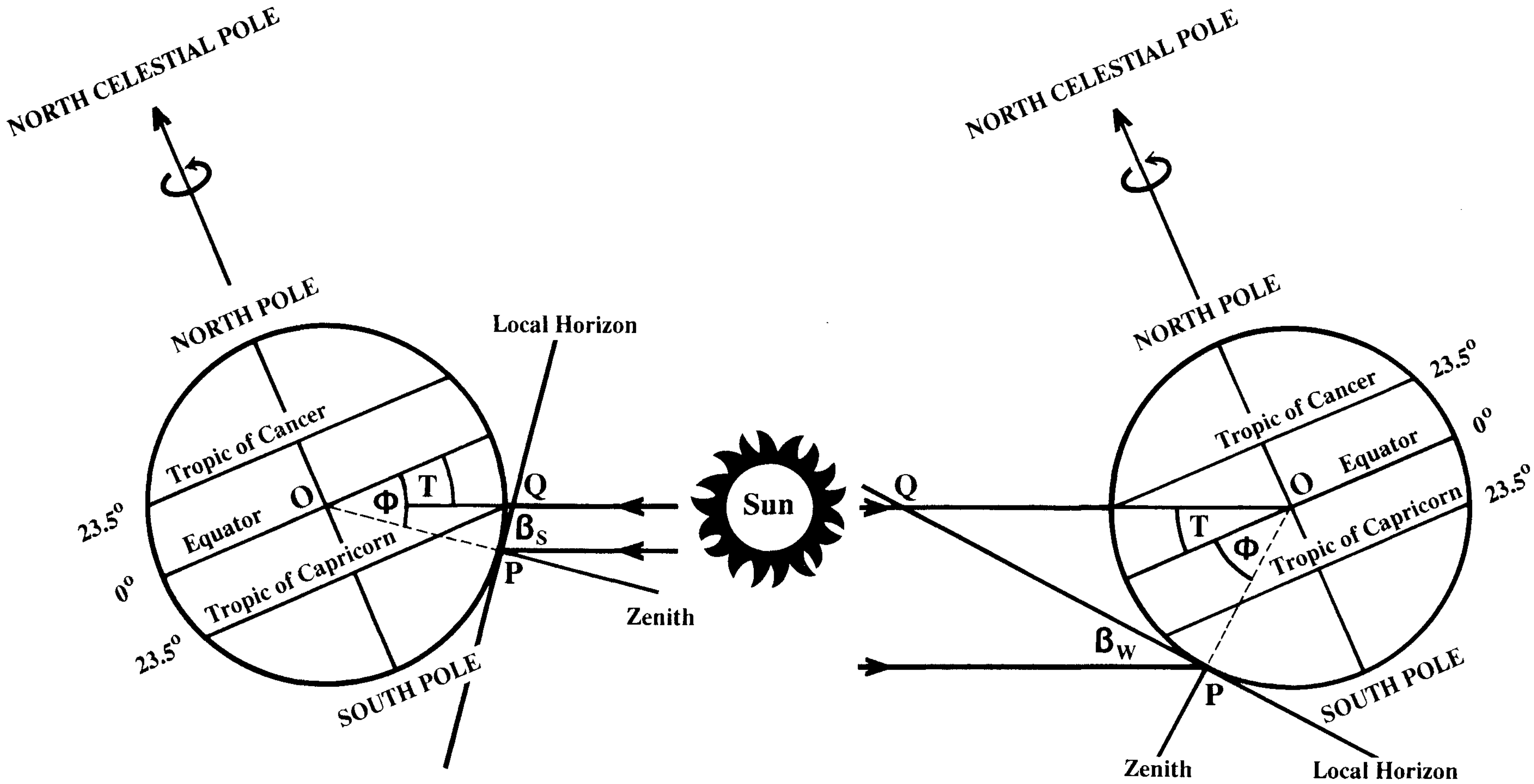
Finally, for the purpose of this text we should mention that the PLANE OF THE ECLIPTIC is inclined to the PLANE OF THE EQUATOR by an angle of approximately 23^{o} 27', known as the OBLIQUITY OF THE ECLIPTIC. These two planes intersect in a line known as the LINE OF NODES and the points where this line of nodes intersects the celestial sphere are known as the EQUINOXES.

The apparent motion of the sun in the ecliptic about the Earth, due to the motion of the Earth in its own orbit, causes the sun to pass through each of the equinoxes once each year. The point where the sun crosses the equator from South to North is known as the VERNAL EQUINOX and the opposite extremity of the line of nodes is called the AUTUMNAL EQUINOX. These definitions have been used historically for many centuries and are strictly applicable to the Northern Hemisphere. It should be remembered that in the Southern Hemisphere these events occur six months out of phase with their Northern counterparts.

Also shown in **Figure 19** are the poles and tropics in their correct positions relative to each other throughout the year as these rotations take place.

To calculate the annual difference between the highest or lowest altitude angle of the sun at solar noon - we should consider what the situation is on midsummers day, and midwinters day for the Southern Hemisphere.

To do this, we must examine in more detail, the extreme left and right hand sides of **Figure 20.**



SOUTHERN HEMISPHERE SUMMERTIME

Summer Solstice, December 22.

The sun is Overhead at the Tropic of Capricorn

SOUTHERN HEMISPHERE WINTERTIME

Winter Solstice, June 22.

The sun is Overhead at the Tropic of Cancer

Figure 20: Altitude of the Sun in Summer and Winter

If we consider **Figure 20**

Let,

P Be a place on the Earth's surface at latitude ϕ .

T Be the tilt angle of the Earth's axis with respect to the plane of its orbit around the sun.

β_s Be the altitude of the sun at solar noon at place 'P', on midsummer's day.

β_w Be the altitude of the sun at solar noon at place 'P' on midwinter's day.

In Δ OPQ **Summer**

$$(\phi - T) + \beta_s + 90 = 180$$

Therefore: $\beta_s = 90 - (\phi - T)$

In Δ OPQ **Winter**

$$(\phi + T) + \beta_w + 90 = 180$$

Therefore: $\beta_w = 90 - (\phi + T)$

From these formulae we can now calculate the sun's altitude angle 'β', at solar noon, for midsummers or midwinters day.

Using the above formulae and substituting the measured dimensions of the sundial time scale gives the following result. Overall the sundial is 1070 x 715 mm and of specific interest in this case is the vertical distance from the gnomon mounting point to the lowest portion of the time scale at 12 o'clock. This value was 780 mm.

Using the above formulae and referring to **Figure 21**, we get the following result.

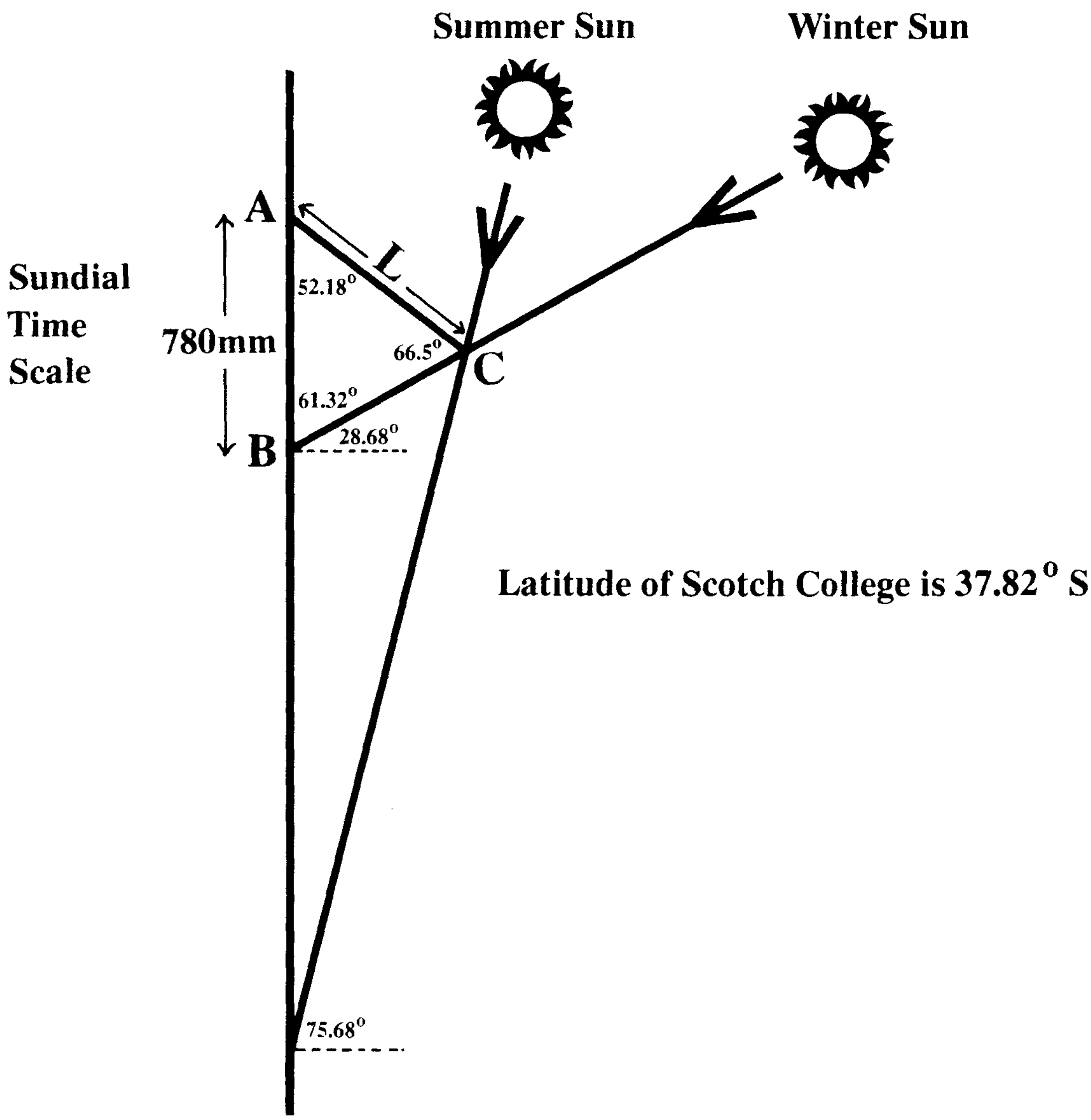


Figure 21: Length of the Gnomon Needed for the Scotch College, Stained Glass Window Sundial

$$\beta_s = 90 - \phi + T \quad \text{Summer}$$
$$= 90 - 37.82 + 23.5$$
$$= \underline{75.68^\circ}$$

$$\beta_w = 90 - \phi - T \quad \text{Winter}$$
$$= 90 - 37.82 - 23.5$$
$$= \underline{28.68^\circ}$$

The noon time altitude of the sun is at its minimum on midwinters day when β_W = 28.68°. To ensure that the gnomon shadow is always longer than the sundial time scale, we refer to Δ ABC, where we note that for midwinter,

$$\frac{L}{\sin 61.32} = \frac{780}{\sin 66.5} \quad \text{and } L = 746.2 \text{ mm}$$

The Minimum Length of the Gnomon is seen to be 746.2 mm.

The gnomon for the Scotch College Sundial was made a few millimetres longer than this value to ensure that the shadow would completely fill the time scale throughout the year.

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*	Dr F G Donaldson	Principal

* Member of Scotch College Staff

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Gnomonists (John Ward)

John Ward

and

(Margaret Folkard)

Margaret Folkard

February 1990

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